Exploring the Quadrupedal Asymmetrical Gaits

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Introduction

Animals in nature adopt a large number of gaits at varying forward speeds to minimize their energy expenditure. In the quadrupedal asymmetrical gaits *bounding* and *galloping*, the front and hind leg pairs move together and touch the ground either at the same time (bounding) or in a quick succession (galloping). The gait is termed as *half-bound* if a lead only occurs in the fore footfalls; and *crutch-walk* if the lead only happens in the hind leg pair [1]. The exact footfall pattern, however, tends to differ from animal to animal. Cheetahs and greyhounds, for example, exhibit two aerial phases within one stride, whereas hopping weasels and deer usually have only one flight phase [1]. Furthermore, the flight phase can occur either after the front leg pair is in contact or after the hind leg pair is in contact [2]. Sometimes gazelles also *pronk*; that is, all four legs hit and leave the ground at exactly the same time, yielding yet another contact sequence [3]. According to the previous work on legged locomotion, when these animals are scaled by their body mass, leg length and gravity, they should share similar dynamic behaviors. Alexander *et al.* [4] found that quadrupedal animals prefer to use the same gait at identical Froude numbers and they also found that various mammals change from symmetrical gaits to asymmetrical gaits at Froude number near 2 or 3. Nevertheless, it is not clear why there are so many asymmetrical gaits in quadrupedal animals and why certain animals choose to use a particular type of asymmetrical gait while a lot more similar options are available.

Methods

In this work, we propose a unified passive quadrupedal model that can reproduce the dynamics of all these asymmetrical gaits. The model has four compliant legs and a rigid body as torso which is an extension of the well-known bipedal SLIP model [5]. In order to reduce the dimensions of the solution manifold and reveal the basic structure of all asymmetrical gaits from this model, instead of using a free parameter *angle of attack* for each leg, we added dynamic leg swing by introducing an additional passive torsional spring with a constant natural oscillation frequency ω_{swing} between the torso and each limb. As the leg springs are modelled with no damping and the feet are modeled with no mass, there are no energy losses associated with this model. Once started, the model thus moves perpetually; –following a motion trajectory that is solely a function of the starting configuration.

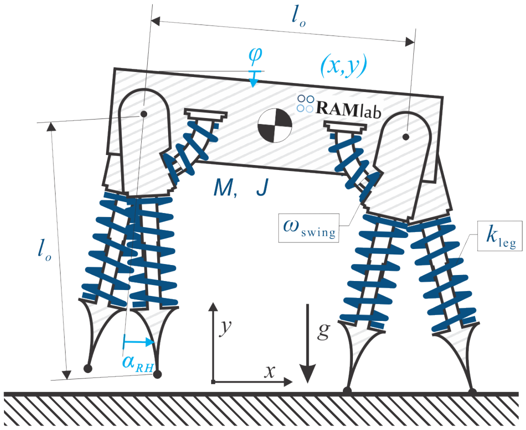

Figure 1: The passive quadrupedal model that is used in this study.

Using numerical continuation and bifurcation theory, we identified all possible periodic motions of this model as they emerge from pronking-in-place. This approach is based on techniques that we originally developed for a similar bipedal model [6]. We found that all gaits from this model are laying on 1-dimensional branches by gradually breaking the symmetry in the solutions.

Results

We conducted the investigation with a leg stiffness of $k_{\text{leg}} = 10\,{}^{Mg}/_{l_o}$, hip springs that resulted in a swing frequency $\omega_{\text{swing}} = \sqrt{5}\sqrt{{}^{g}/_{l_o}}$ and main body inertia $J = 1\,\mathrm{Ml}_o^2$. Our approach was able to show that bounding (2-beat gait), half-bound/crutch-walk (3-beat gait) and galloping (4-beat gait) all emerged from the passive dynamics of the model. As illustrated in Fig. 2, for different initial conditions of the system the passive model can reproduce all the different footfall sequences of asymmetrical gaits found in nature. By varying the total energy in the system E_{tot}, the dynamic be-

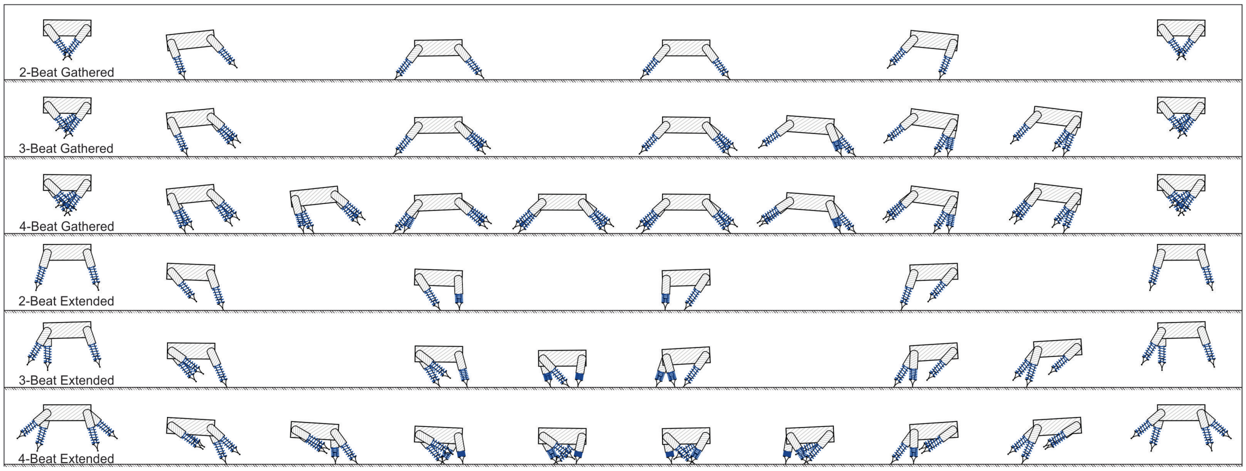

Figure 2: Successive motion frames of the asymmetrical gaits found in a purely passive model: bounding (2-beat gait with gathered/extended suspensions), half-bound/crutch-walk (3-beat gaits with gathered/extended suspensions), and gallop (4-beat gaits with gathered/extended suspensions).

havior of the system could be influenced. In particular, the bounding were found as pitch-fork bifurcations of the pronking. The half-bound and crutch-walk all emerged from the 2-beat bounding gait. Furthermore, we have identified that the galloping branch originates from the branches of 3-beat gaits.

Discussion

Our work shows that all instances of asymmetrical gaits can be generated with a single passive model and a single set of parameters. In a sense, the different gaits are just different oscillation modes of a single mechanical framework. These solutions are highly dependent on the total energy that includes the kinetic energy, gravitational potential energy, and the spring potential energy stored in compliant stance legs. As we vary the system energy, there are always finite number of asymmetrical gaits we can find from this model. From the pronking gait, by introducing the main body rotation the symmetry in the frontal plan is lost and two bounding gaits with gathered and extended suspensions appear. When a pair of legs desynchronises and strikes the ground in a sequence, the symmetry in the sagittal plan breaks and then all the 3-beat and 4-beat gait can be found.

Moreover, we have found that the main body inertia J has a huge influence on these gaits. In particular, decreasing the inertia joined some of solutions branches with different footfall patterns. For very low inertia values, some of the footfall patterns vanished at higher speeds. These results could shed light on the underlying cause for certain gait choices and could be used as templates in the design of legged robotic systems.

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