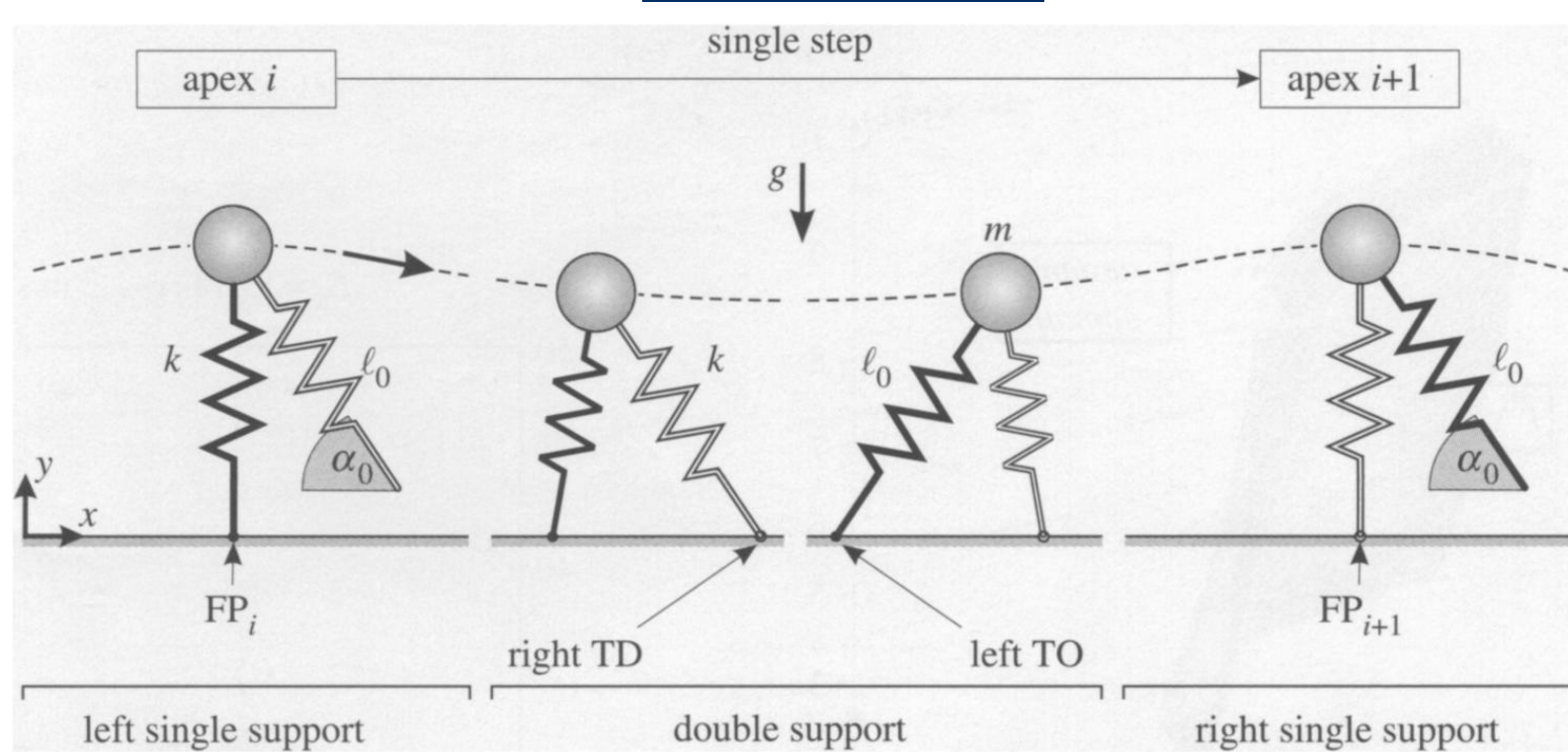


Motivation

SLIP Model



- Bipedal Spring Loaded Inverted Pendulum (SLIP) model with one predefined contact angle (angle of attack) for both legs can be used to simulate *walking* and *running* gaits.
- Touch-down and lift-off events are triggered when foot strikes the ground (event detection functions).
- There are other bipedal gaits such as *hopping*, *skipping*, and *galloping*, where two legs can have different angles of attack.
- Numerical errors are observed when foot is hovering only slightly above/below the ground (for instance, *grounded running*).

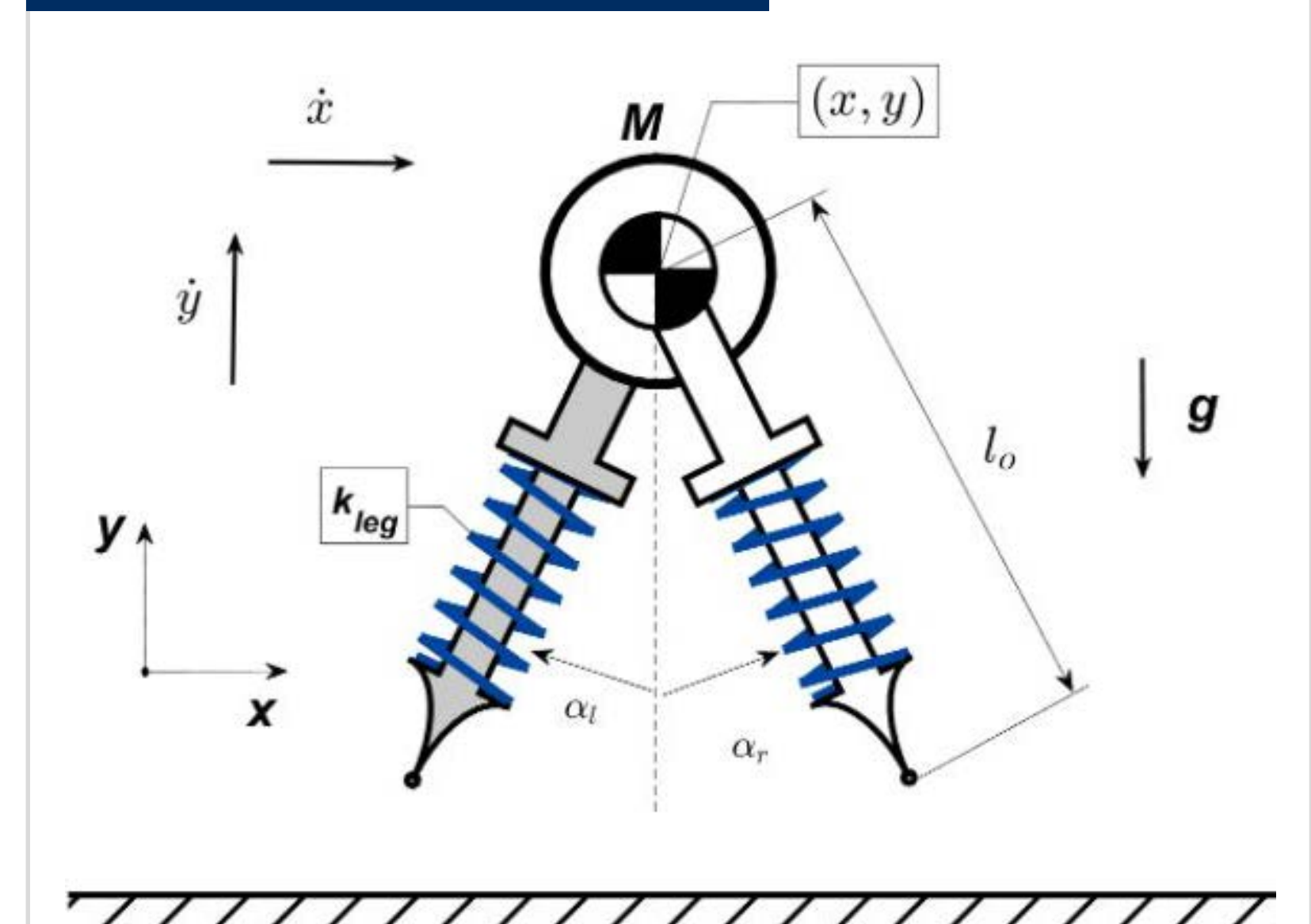
○ There is a need to develop a model that can find all bipedal gaits, and a better way to define events is required.

Gait Creation

- The Gait Creation is the implementation of Boundary Value Problem (BVP).
- The generalized position and velocity in the model are $\mathbf{q} = [x, y]^T$ and $\dot{\mathbf{q}} = [\dot{x}, \dot{y}]^T$.
- Except for *asymmetrical walking*, Poincare section is selected in apex where $\dot{y} = 0$.
- To avoid numerical errors that are observed in previous research, four timing variables $\mathbf{e} = [t_{LTD}, t_{LLO}, t_{RTD}, t_{RLO}]^T$ are used to determine the touch-down and lift-off events.
- To find solutions with arbitrary footfall patterns, the order of these events is not specified, but their values are restricted within the time interval of a stride $[0, t_{stride}]$.
- The active parameters \mathbf{p} in the system include:
the total energy E_{total} , and two angles of attack (α_l, α_r).
- The dynamics of the model are governed by a set of differential equations (EOM):
 $\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{e}, \mathbf{p})$.

Model

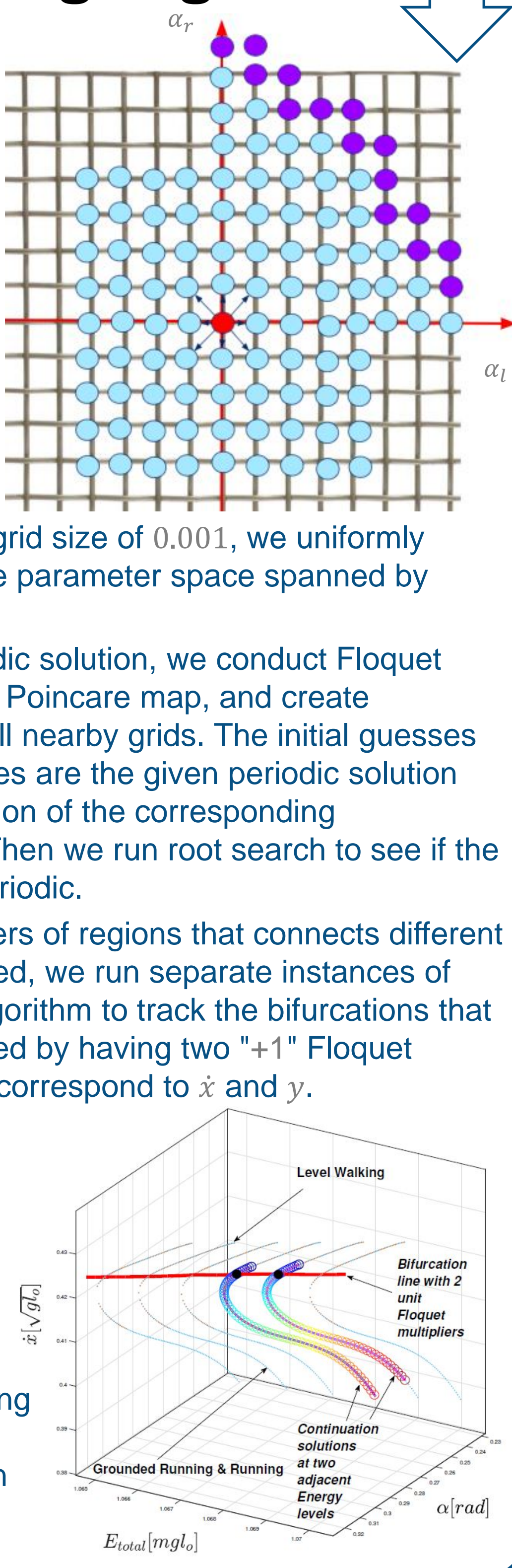
Extended SLIP Model



- To find all bipedal gaits that can be generated passively, we modified the original bipedal SLIP model by including two varying angles of attack (α_l, α_r) for two legs.
- Neither friction nor collision losses are considered so that an energetically conservative system is created.
- Angles of attack are periodic.
- Fixed parameters in this study: Spring stiffness $k_{leg} = 20$, the gravity $g = 1$, the rest length for both legs $l_0 = 1$, and the center of mass (COM) $M = 1$.

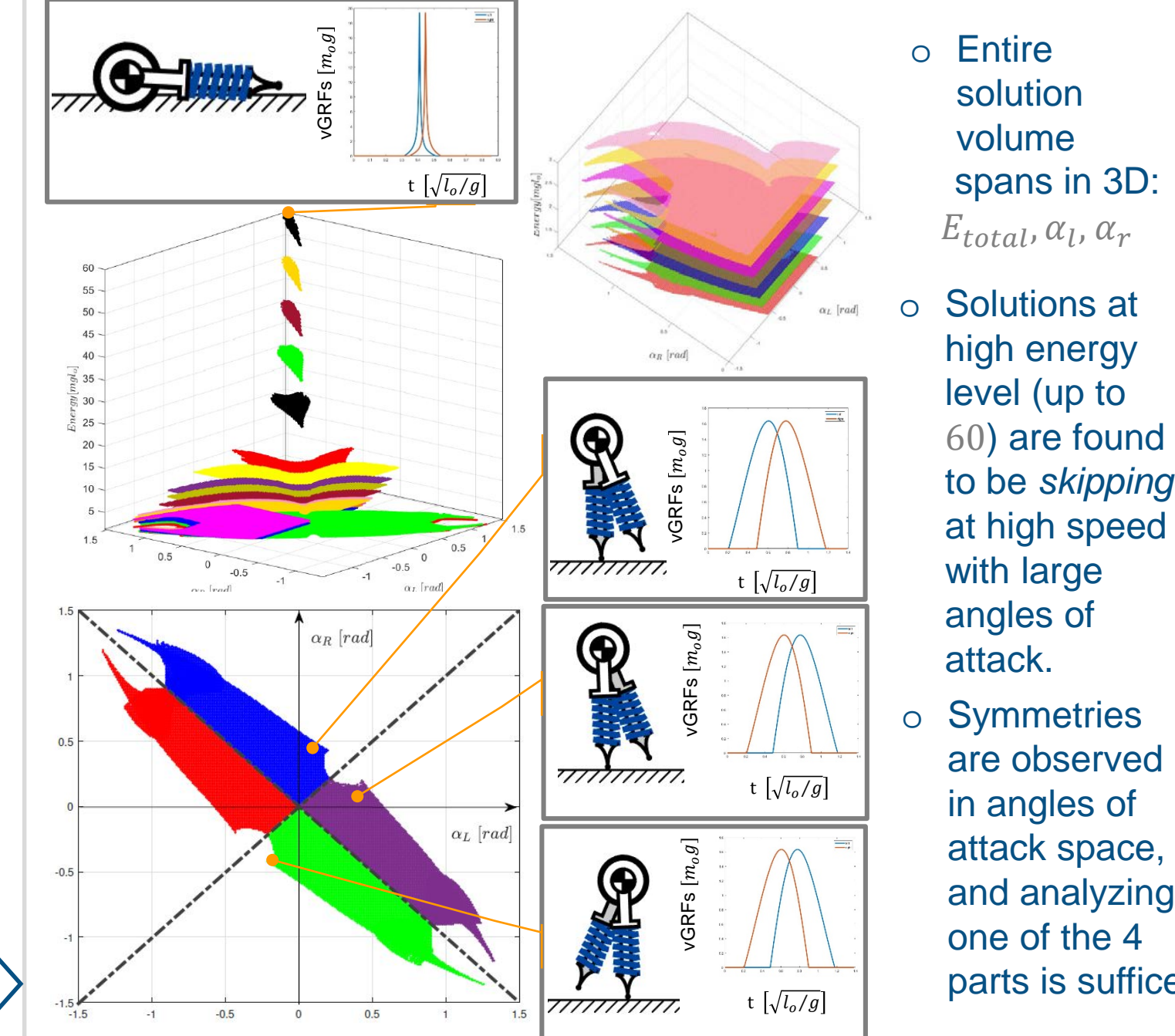
Searching Algorithm

- To identify all regions of periodic solutions from this model, we have conducted a multi-dimensional continuation in the parameter space.
- That is, with a grid size of 0.001, we uniformly mesh the whole parameter space spanned by $E_{total}, \alpha_l, \alpha_r$.
- For each periodic solution, we conduct Floquet analysis on the Poincare map, and create candidates in all nearby grids. The initial guesses of the candidates are the given periodic solution and the prediction of the corresponding eigenvectors. Then we run root search to see if the candidate is periodic.
- When the borders of regions that connects different gaits are reached, we run separate instances of continuation algorithm to track the bifurcations that are characterized by having two "+1" Floquet multipliers that correspond to \dot{x} and y .
- The model and the searching algorithm are implemented in MATLAB. The root search is solved numerically using the integrator ODE45, with an accuracy of 10^{-9} .



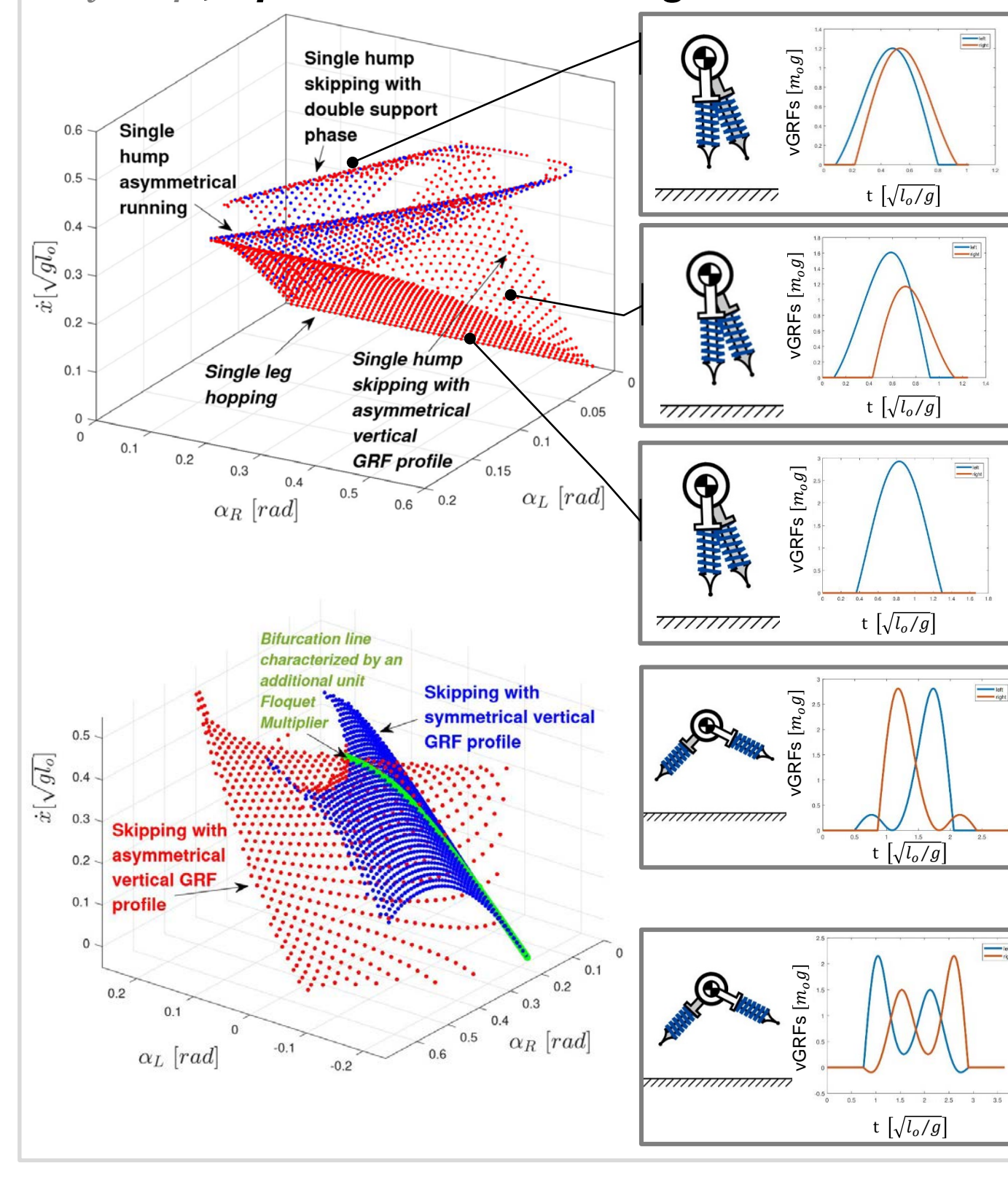
Current Results

Solution Volume & Searching Scope



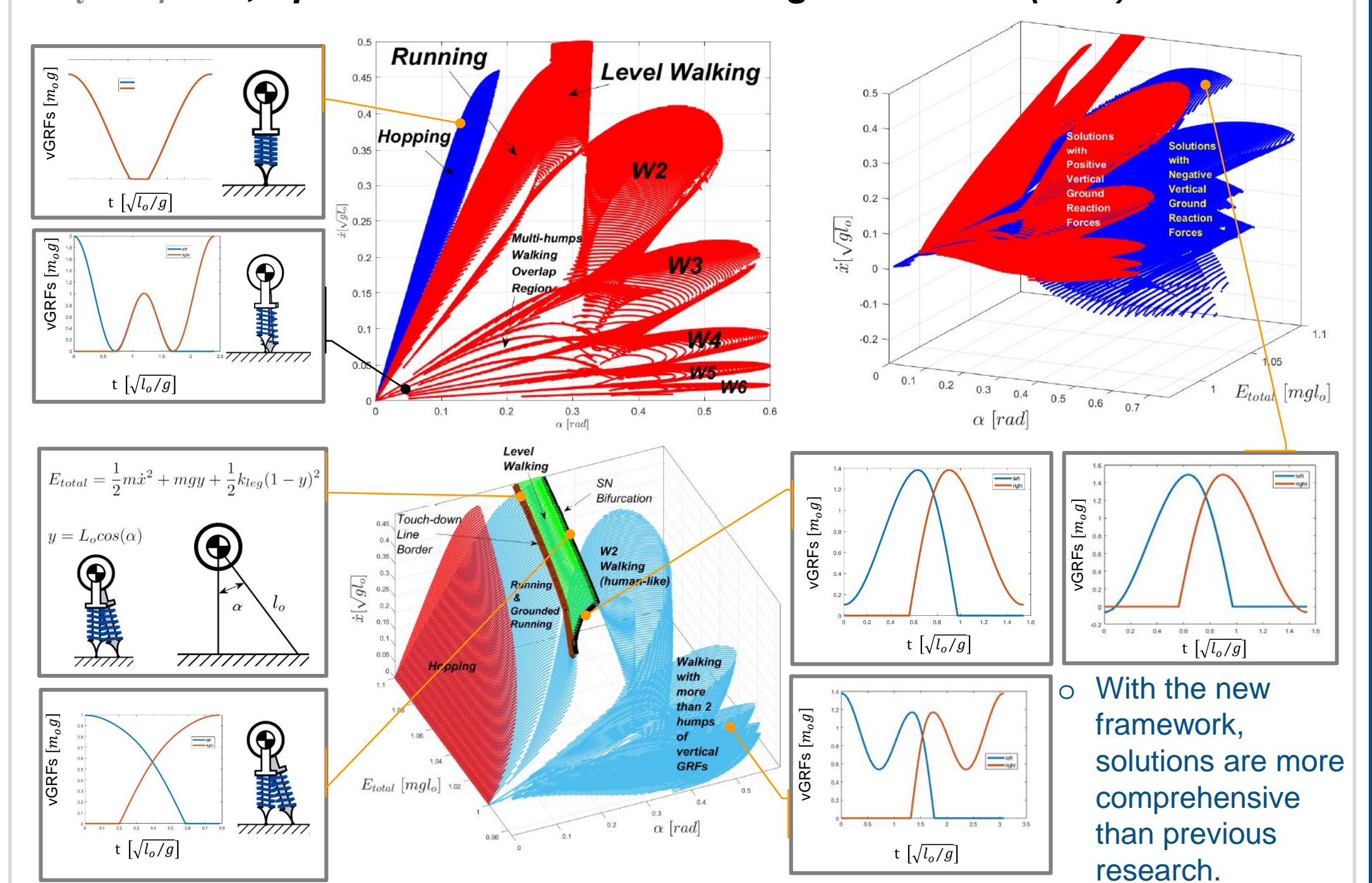
- Entire solution volume spans in 3D: $E_{total}, \alpha_l, \alpha_r$
- Solutions at high energy level (up to 60) are found to be *skipping* at high speed with large angles of attack.
- Symmetries are observed in angles of attack space, and analyzing one of the 4 parts is suffice.

$\alpha_l \neq \alpha_r$, Apex Transition in Flight



- By enforcing the following symmetry in angles of attack, we find solutions with symmetrical GRFs profiles.
 $||\alpha_{rTD}|| = ||\alpha_{lLO}||$
 $||\alpha_{lTD}|| = ||\alpha_{rLO}||$

$\alpha_l = \alpha_r = \alpha$, Apex Transition in Vertical Leg Orientation (VLO)



$\alpha_l \neq \alpha_r$, VLO

