Phase Synchronization of Compliant Legs using Virtual Damping Control

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Introduction

Problem Definition

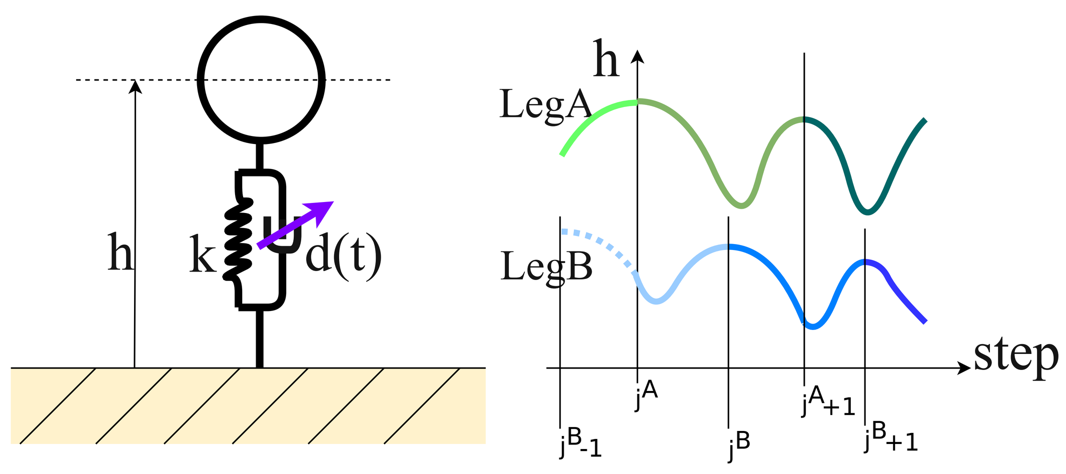
- Simplified monopedal spring-mass models can capture the dynamics of locomotion
- Physical robots, however often incorporate multiple legs to overcome actuator limitations, improve stability and support on terrain of varying roughness
- A common problem here is to ensure a particular phase relation between oscillations of different legs for different locomotory gaits

Contributions

- We propose a new method to achieve synchronization patterns using virtually tunable damping coefficient on individual legs, resulting in energy and power efficiency
- We show that a hybrid leg model with intermittent contact offers better affordance over system phase even when only the damping coefficient is explicitly controlled

Synchronization Framework

Dynamic System Model



State vector for a single leg defined as

$$\mathbf{x}^i := \begin{bmatrix} h^i & \dot{h}^i \end{bmatrix}^T$$

where legs are labelled with $i \in A, B$. The same structure will be used for a continuous Spring-Mass-Damper (SMD) and a Spring-Loaded Inverted Pendulum (SLIP) model, both with tunable damping.

The Effects of Leg Damping

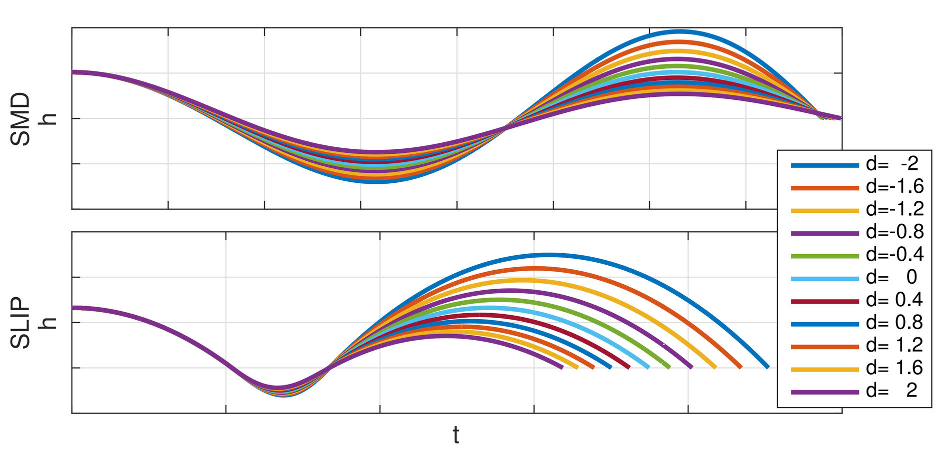


Figure: SMD and SLIP reaction over damping.

- Changing $d(t)$ for the SMD model has very little effect on the phase
- Damping stance for the SLIP model has substantial effect on the timing for the next stride and hence the phase.

Return Map Analysis

We can merge the state spaces of two legs as $\mathbf{x}^A \in \mathcal{X}^A, \mathbf{x}^B \in \mathcal{X}^B \to \mathbf{z} \in \mathcal{Z} := \mathcal{X}^A \otimes \mathcal{X}^B$, yielding

$$\dot{\mathbf{z}} = f_x(\mathbf{z}, \mathbf{u}) \,.$$

Limit cycles can be found with a Poincaré section $\mathcal{S} := \{\mathbf{z} \in \mathcal{Z} \,|\, \dot{h}^A = 0\} \subset \mathcal{Z}$ at apex states.

Synchronization Framework

Phase and Energy Coordinates

The equations of motion for the SLIP model are defined for stance and flight as

$$\begin{bmatrix} \dot{h} \\ \ddot{h} \end{bmatrix}_{st} = \begin{bmatrix} 0 & 1 \\ -k & -d(t) \end{bmatrix} \begin{bmatrix} h \\ \dot{h} \end{bmatrix}, \quad \ddot{h}_{fl} = -g \,.$$

As in [1], a coordinate transformation is introduced with $\mathbf{y} = \mathbf{W}\mathbf{x}$, by using the matrix

$$\mathbf{W} := \begin{bmatrix} \omega\sqrt{1+\beta^2} & \frac{\beta}{\sqrt{1+\beta^2}} \\ 0 & \frac{1}{\sqrt{1+\beta^2}} \end{bmatrix}, \tag{1}$$

where we define

$$\omega := \frac{\sqrt{4k - d^2}}{2}, \quad \beta := \frac{d}{2\omega}.$$

This transforms the system into the canonical form (for the SMD model)

$$\dot{\mathbf{y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \omega \begin{bmatrix} -\beta & 1 \\ -1 & -\beta \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{2}$$

During both stance and flight, leg phase can be defined as

$$\phi := tan^{-1}(\frac{y_2}{y_1}) \,. \tag{3}$$

The evolution of leg phase during stance and flight are respectively given by

$$\dot{\phi}_{st} = -\omega \,; \tag{4}$$

$$\dot{\phi}_{fl} = -\omega \frac{\left(gh + \dot{h}^2\right)}{kh^2 + \dot{h}^2 + dh\dot{h}} \,. \tag{5}$$

The energy needs to be defined differently for stance and flight, with

$$E_{st} := \frac{1}{2}\left(y_1^2 + y_2^2\right) = \frac{1}{2}\left(kh^2 + \dot{h}^2 + dh\dot{h}\right) \,; \tag{6}$$

and

$$E_{fl} := gh + \frac{1}{2}\dot{h}^2 \tag{7}$$

respectively.

The evolution of this energy coordinate for stance and flight are given by

$$\dot{E}_{st} = -dE_{st}, \qquad \dot{E}_{fl} = 0 \,. \tag{8}$$

Stride-to-Stride Behavior

Combing two legs, we have the joint state vector in phase/energy coordinates defined as

$$\bar{\mathbf{q}} := \begin{bmatrix} \phi^A & E^A & \phi^B & E^B \end{bmatrix}^T \,. \tag{9}$$

Choosing a Poincaré section with $\phi^A = 0$, at the apex point of the first leg, the Poincaré state takes the form

$$\tilde{\mathbf{q}} = \begin{bmatrix} E^A & \phi^B & E^B \end{bmatrix}^T \,. \tag{10}$$

An important issue is that two subsequent strides of Leg B occur within a single stride of Leg A, using two separate damping coefficients. To ensure continuity, the damping coefficient from the previous stride for Leg B needs to be recorded in the Poincaré state, resulting in the definition of state

$$\mathbf{q}_{j^i} := [\Delta\theta_{j^i}, \Delta E^A_{j^i}, \Delta E^B_{j^i}, d^B_{j^i}]^T \,. \tag{11}$$

Before each stride for Leg A, a control decision must be made with

$$\mathbf{u}_{j^i+1} = [d^A_{j^i}, d^B_{j^i}]^T = f(\mathbf{q}_{j^i}) \tag{12}$$

with the goal of reaching equilibrium at

$$\mathbf{q}_{j^i+1} = R(\mathbf{q}_{j^i}) = \mathbf{q}^* = [0, 0, 0, 0]^T \,. \tag{13}$$

Controller Design

Control of phase and energy accomplished by modulating damping coefficients for both legs, using the proportional feedback laws

$$d^A_{j^A+1} = \alpha^A_E\left(E^A_{j^A} - (E^* + \alpha^A_\phi \Delta\phi_{j^A})\right) \tag{14}$$

$$d^B_{j^B+1} = \alpha^B_E\left(E^B_{j^B} - (E^* - \alpha^B_\phi \Delta\phi_{j^B})\right) \,. \tag{15}$$

In our results, we use $\alpha^A_\theta = 0$, which selects Leg A as the "phase master", concerned with its own energy alone. Leg B, on the other hand, regulates both its own energy as well as its phase relationship with Leg A.

Simulation Results

For this simulation, controller parameters are selected as $\alpha^A_\phi = 0, \alpha^B_\phi = 1, \alpha^A_E = \alpha^B_E = 0.1$ and desired states are $\theta^* = \pi/2$, $E^* = 15$

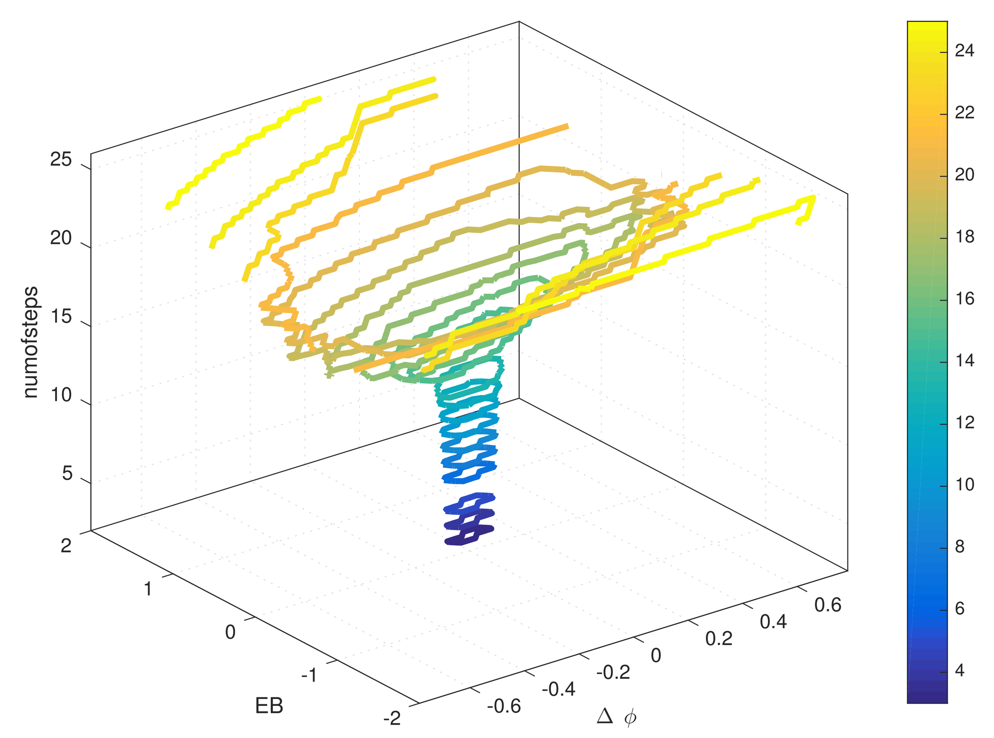


Figure: Convergence to the desired state from varying initial conditions within the tolerance of error 0.1.

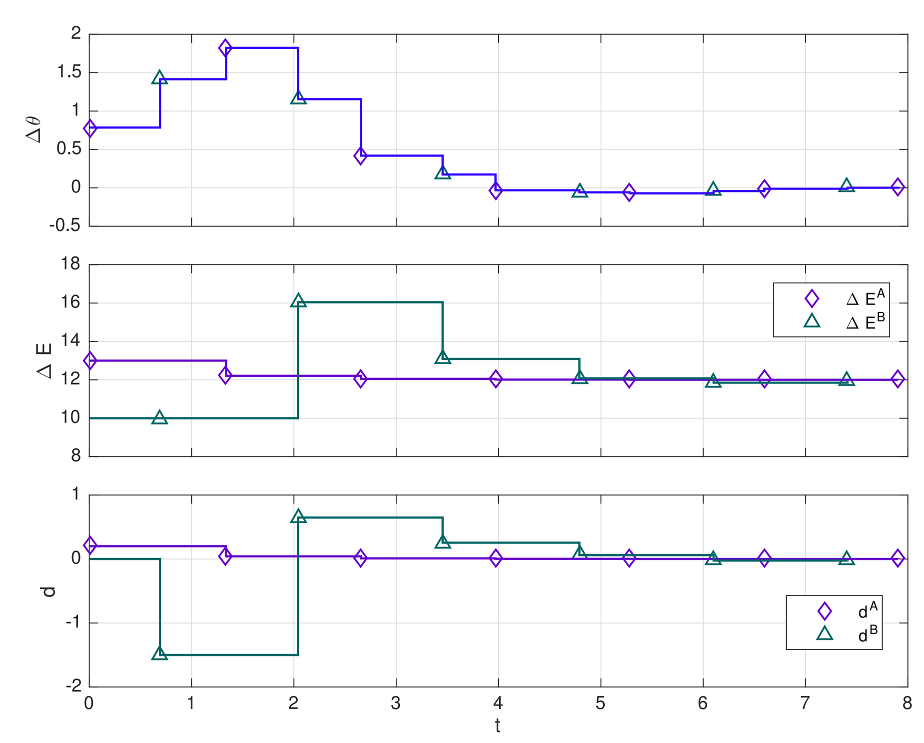


Figure: Phase difference,energy error and damping coefficients for a sample run.

Conclusion and Future Work

- Phase synchronization of compliant legs via damping control was attempted for the SMD model, but damping offers very little affordance over the SMD phase.
- Synchronization was achieved with the SLIP model through a proportional controller.
- Convergence properties were investigated in simulation, with more elaborate analysis left as future work.

Acknowledgement

This work was supported by TUBITAK project 117E106 as well as Merve Özen's scholarship. We also thank Görkem Seçer for presenting this poster in conference.

Further reading

[1] Eric Klavins and Daniel E Koditschek. Phase regulation of decentralized cyclic robotic systems. *The International Journal of Robotics Research*, 21(3):257–275, 2002.