Exploring the Quadrupedal Asymmetrical Gaits

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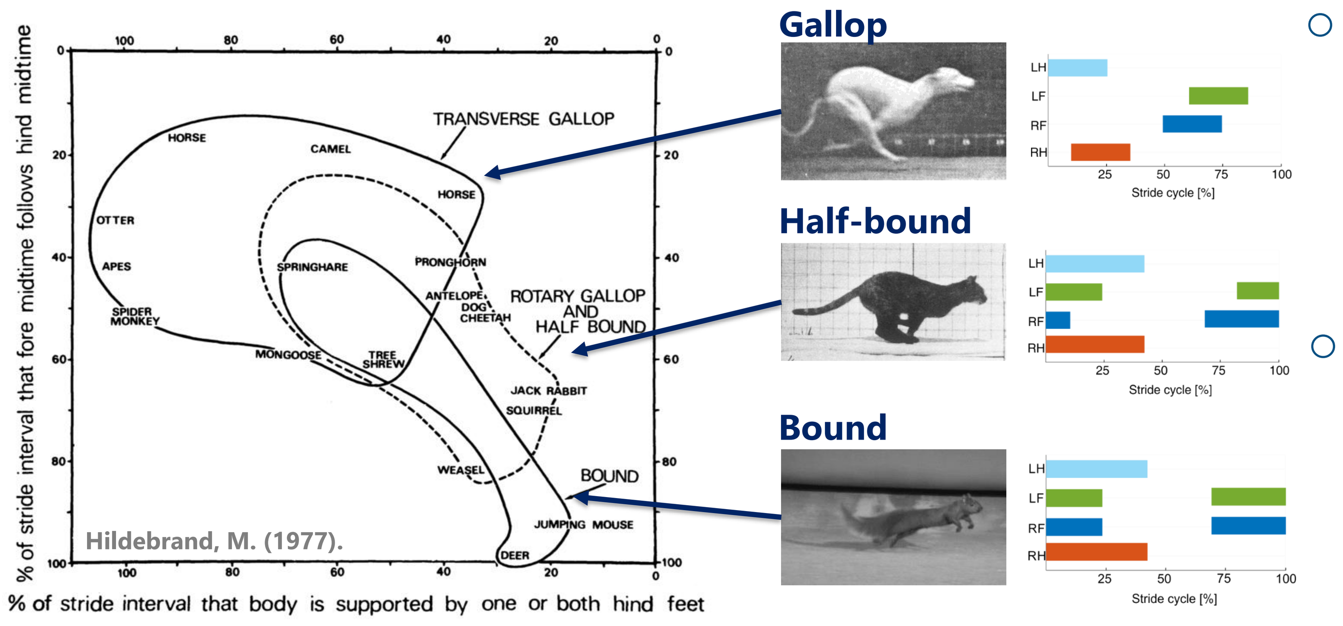
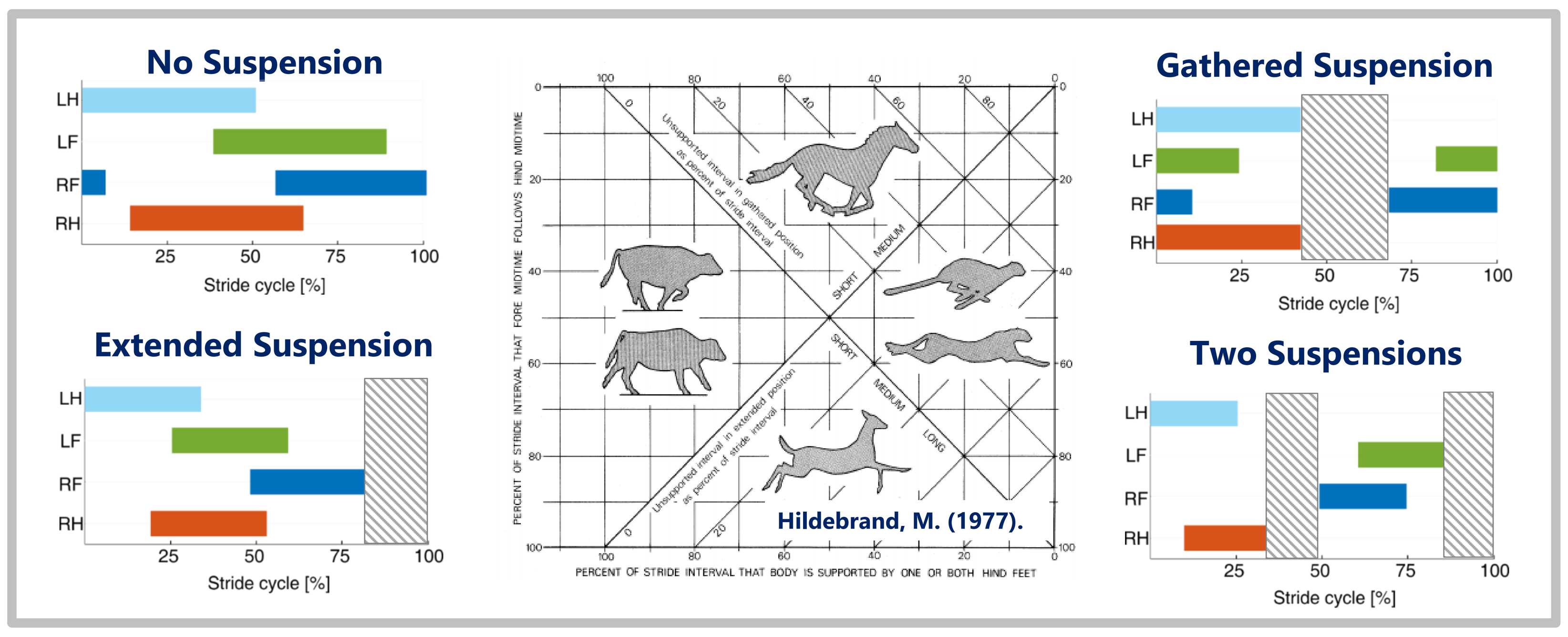
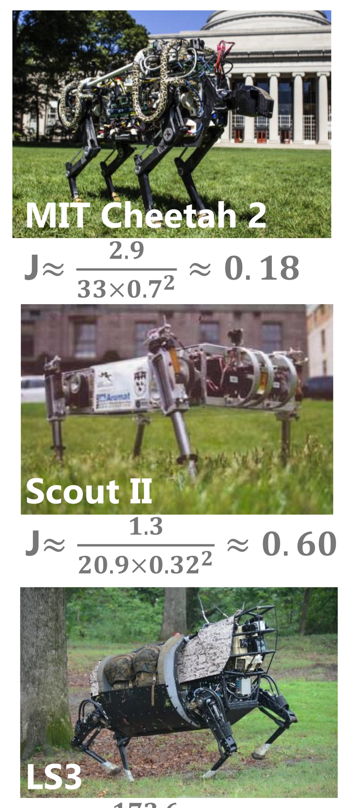
Motivation



- There exist a large number of quadrupedal asymmetrical gaits according to their footfall patterns and the flight phases
- Which gait should we use on different robots at a specific speed?



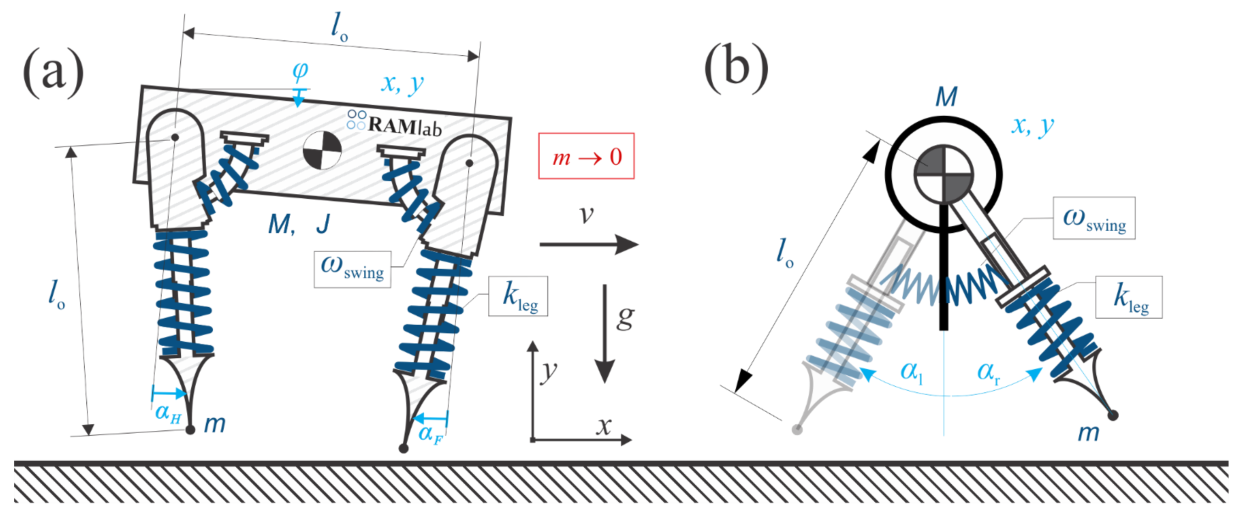


$J \approx \frac{2.9}{33 \times 0.7^2} \approx 0.18$

$J \approx \frac{1.3}{20.9 \times 0.32^2} \approx 0.60$

$J \approx \frac{173.6}{590 \times 0.55^2} \approx 0.97$

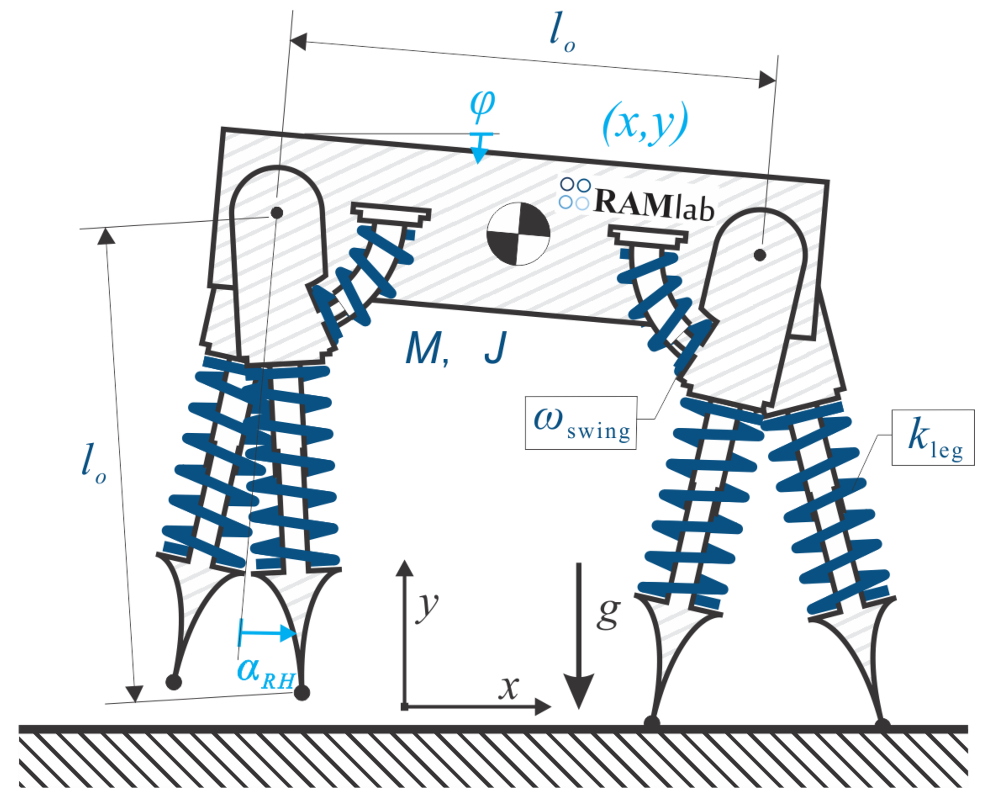
Model



- To find the unique passive gait, we modified the conventional SLIP model [2] by adding a torsional spring at the hip and fix the frequency of the torsional swing leg spring during the study:

$$\omega_{swing} = \sqrt{\frac{k_{swing}}{ml_o^2}}$$

- Replace k_{swing} with $\left(\frac{k_{swing}}{m}\right) m$, then we take the limit of $m \to 0$ to preserve non-trivial swing dynamics.
- Avoid the collision losses at touch down, keep system conservative



Continuation

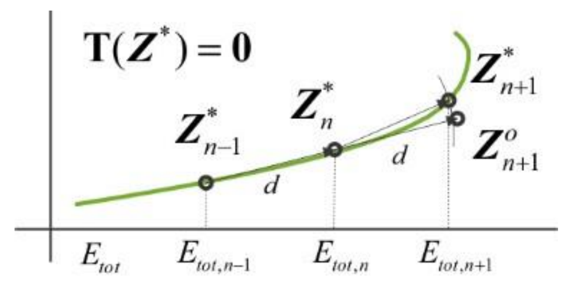
- To find a gait with the specific footfall pattern, we introduce eight timing variables e for touchdown and liftoff events that breaks the stride into 9 intervals.
- Similar to [3] [4], we stated the gait creation as a boundary value problem (BVP):

$$\mathrm{T}(\boldsymbol{Z}^*) := \begin{bmatrix} \ddot{\boldsymbol{q}} - \mathrm{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t, \boldsymbol{e}) \\ \dot{\boldsymbol{q}}(t_k^+) - h(\boldsymbol{q}(t_k^-), \dot{\boldsymbol{q}}(t_k^-)) \\ \mathrm{R}_{1\text{-}15}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{e}, t_{stride}) \end{bmatrix} = \mathbf{0}.$$

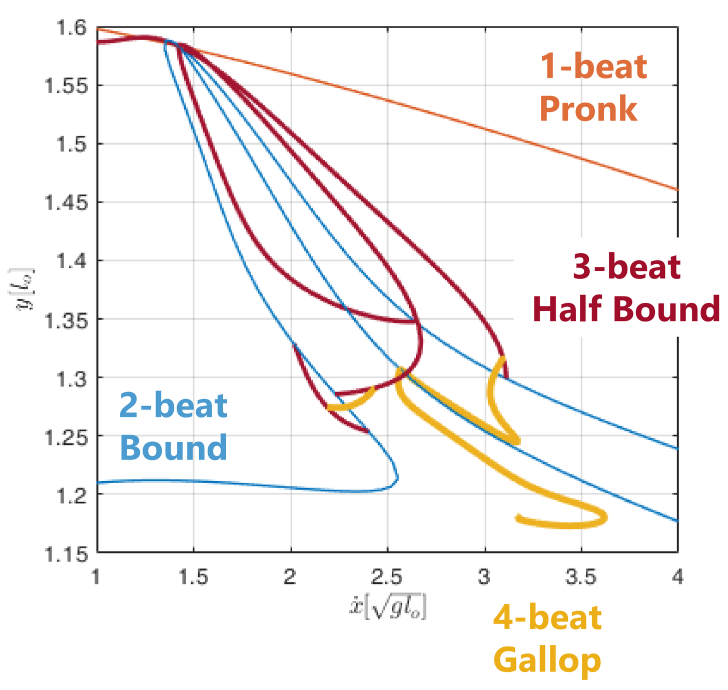
R includes the boundary and interior-point conditions, for example periodicity of states and foot positions at events.

- Then we iteratively solving the continuation problem using arc-length parameterization:

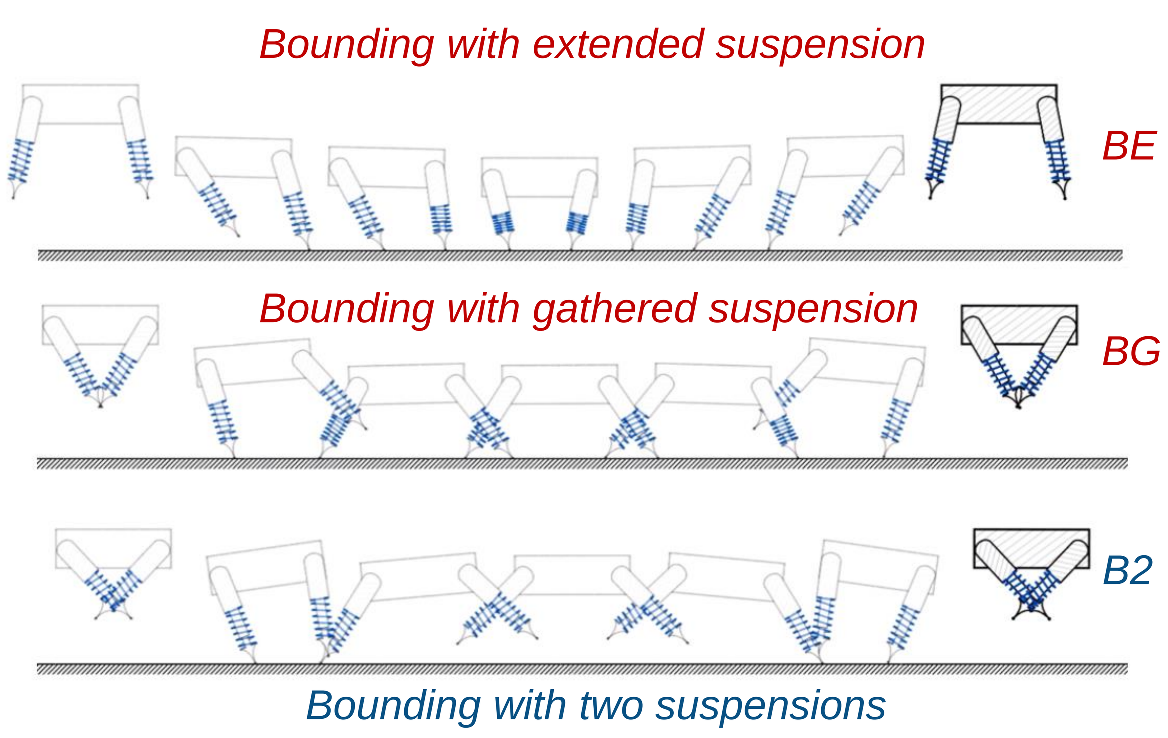
$$\mathrm{T}(\boldsymbol{Z}^*_{n+1}, E_{tot,n+1}) = \mathbf{0},$$
$$\|\boldsymbol{Z}^*_{n+1} - \boldsymbol{Z}^*_n\| = d,$$
$$(\boldsymbol{Z}^*_{n+1} - \boldsymbol{Z}^*_n)^T (\boldsymbol{Z}^*_n - \boldsymbol{Z}^*_{n-1}) > 0.$$

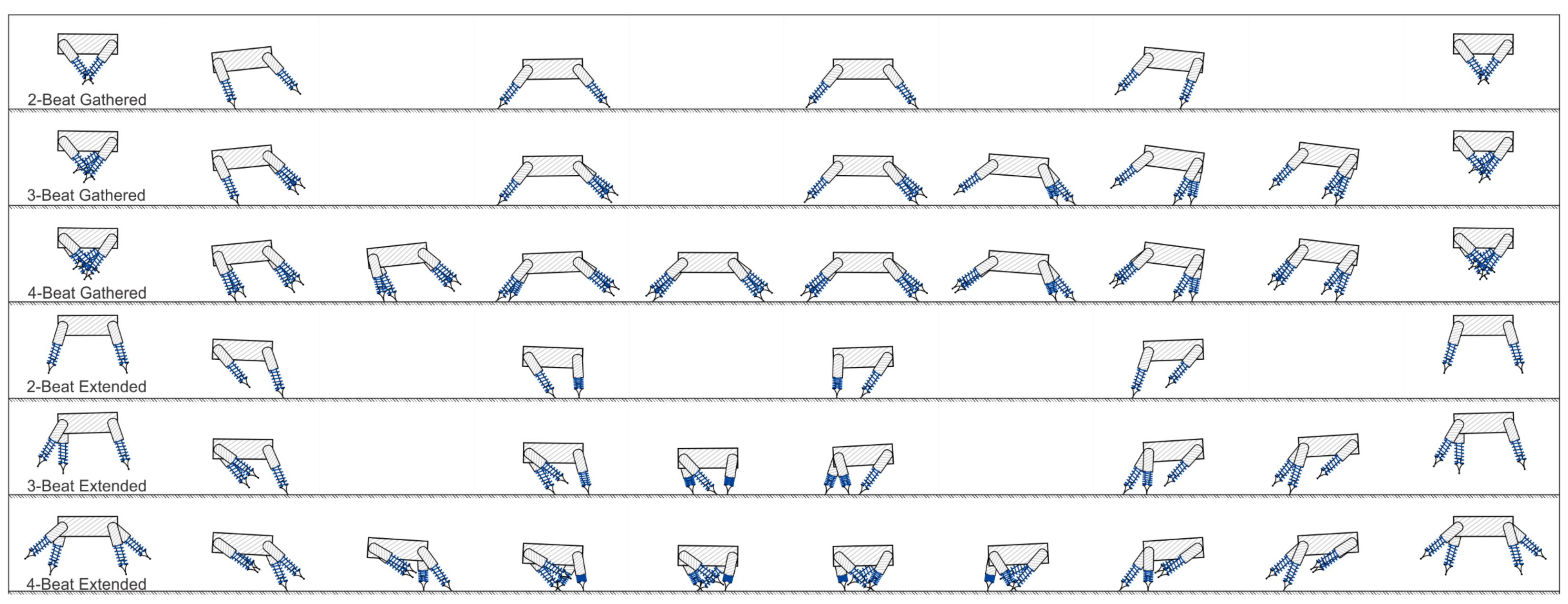


Current Results

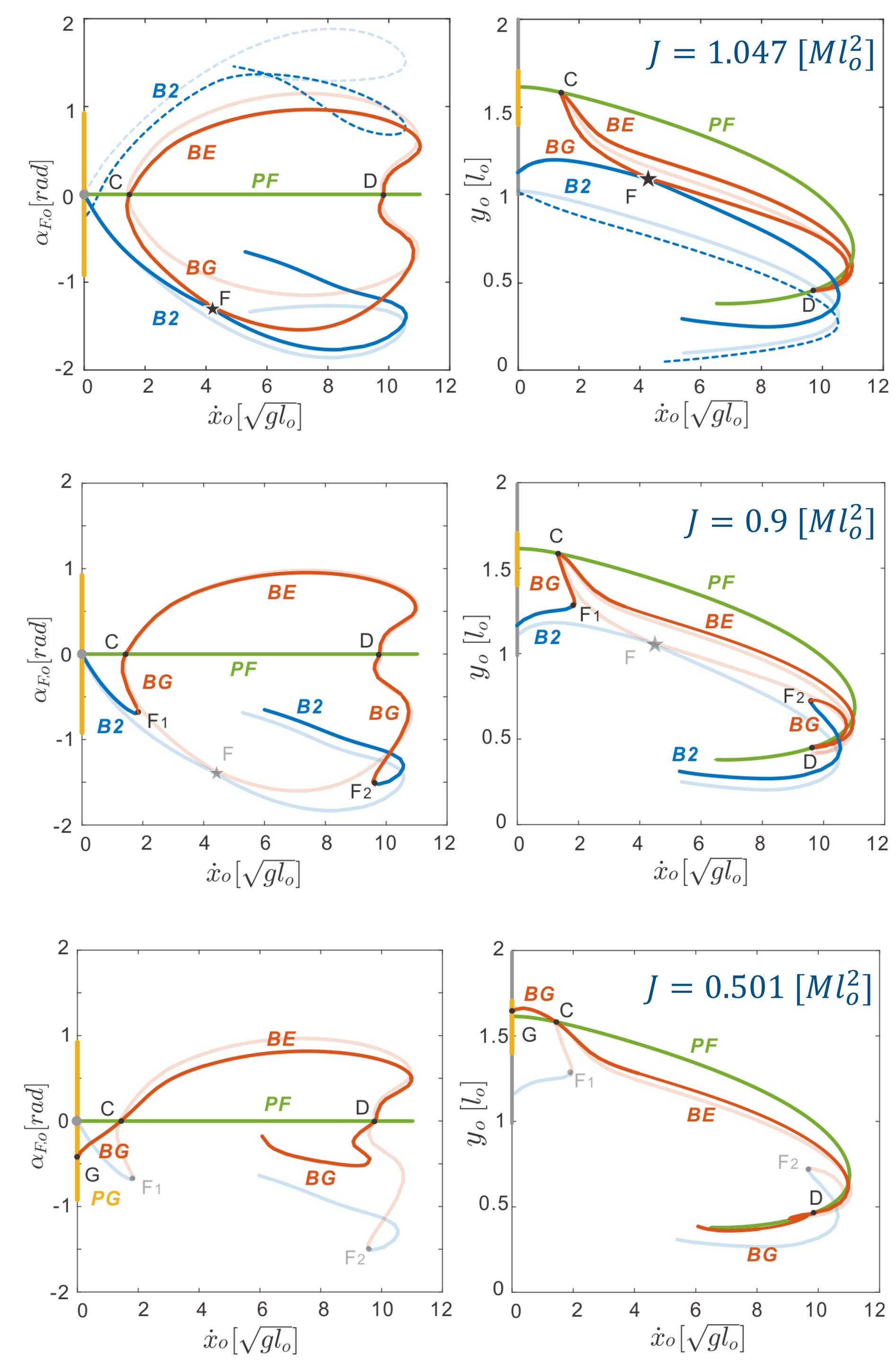


- Different quadrupedal asymmetrical gaits emerge as different types of bifurcations of the same mechanical system from in-place motions.
- These motions are completely passive and may serve as a template to develop energetically economical motions for legged robotic systems.





- The bounding branch with gathered suspension (*BG*) [5] joins branch *B2* at point *F* when the main body inertia *J* reaches to 1.047 $[Ml_o^2]$.
- The bounding gait with two suspensions (branch *B2*) ceases to exist passively when the main body inertia *J* drops below 0.501 $[Ml_o^2]$



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[1] M. Hildebrand, “Analysis of asymmetrical gaits,” *Am. Soc. Mammal.*, vol. 58, no. 2, pp. 131–156, 1977.
[2] Z. Gan, Y. Yesilevskiy, P. Zaytsev, and C. D. Remy, “All Common Bipedal Gaits Emerge from a Single Passive Model,” *Int. J. Rob. Res.*, p. under review, 2017.
[3] M. Hermann and M. Saravi, *Nonlinear Ordinary Differential Equations*. New Delhi: Springer India, 2016.
[4] A. Merker, “Numerical bifurcation analysis of the bipedal spring-mass model,” 2014.
[5] Z. Gan, Z. Jiao, and C. D. Remy, “Dynamics Similarity between Bipeds and Quadrupeds: a Case Study on Bounding,” *IEEE Robot. Autom. Lett.*, p. under review, 2018.

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